



WEST BENGAL STATE UNIVERSITY
B.Sc. Honours/Programme 3rd Semester Examination, 2021-22

MTMHGEC03T/MTMGCOR03T-MATHEMATICS (GE3/DSC3)

REAL ANALYSIS

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any **five** questions from the following: 2×5 = 10
 - (a) Find the least upper bound of the set $S = \left\{ \frac{1}{p} + \frac{1}{q} : p, q \in \mathbb{N} \right\}$.
 - (b) Prove that $\mathbb{N}$ is not bounded above.
 - (c) Show that 0 is a cluster point of the set $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$.
 - (d) Show that $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$.
 - (e) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$.
 - (f) Examine whether the sequence of functions $\{f_n\}$ converges uniformly on $\mathbb{R}$, where for all $n \in \mathbb{N}$,

$$f_n(x) = \frac{x}{n}, \text{ for all } x \in \mathbb{R}.$$
 - (g) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^3 + n^4 x^2}$ is uniformly convergent on $\mathbb{R}$.
 - (h) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n} x^n$.
 - (i) Show that the sequence $\left\{ \frac{1}{n} \right\}$ is a Cauchy sequence.
2. (a) If S is a non empty subset of $\mathbb{R}$ and also bounded below then prove that S has an infimum. 3
 - (b) Show that the subset $S = \{x \in \mathbb{Q} : x > 0, x^2 < 2\}$ is a non empty subset of $\mathbb{Q}$, bounded below; but $\inf S$ does not belong to $\mathbb{Q}$. 5
3. (a) Show that 0 is a limit point of the set $\{x : 0 < x < 1\}$. 2
 - (b) Find all limit points of the set of all rational numbers $\mathbb{Q}$. 3
 - (c) Prove that $\mathbb{Z}$ is not bounded below. 3
4. (a) Prove that the set of all open intervals having rational end points is enumerable. 4
 - (b) Show that the sequence $\left\{ \frac{n^2 + 2022}{n^2} \right\}$ converges to 1. 4

5. (a) Show that the sequence $\{x_n\}$ is monotone increasing, where 4

$$x_n = 1 + \frac{1}{2} + \dots + \frac{1}{n} \quad \text{for all } n \in \mathbb{N}$$

Hence show that the sequence $\{x_n\}$ is not convergent.

- (b) Apply Cauchy's criterion for convergence to show that the sequence $\{x_n\}$ is convergent, where 4

$$x_n = 1 + \frac{1}{2!} + \dots + \frac{1}{n!} \quad \forall n \in \mathbb{N}$$

6. (a) Let $x \in \mathbb{R}$. Show that the series $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ converges absolutely. 4

- (b) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)}$. 4

7. (a) Discuss the convergence of the series $\sum 1/n^p$, $p > 0$. 4

- (b) Let $f_n(x) = x^n$, $x \in [0, 1]$. Show that the sequence of function $\{f_n\}$ is not uniformly convergent on $[0, 1]$. 4

8. (a) Let $f_n(x) = nxe^{-nx^2}$, $x \in [0, 1]$, $n \in \mathbb{N}$. Show that the sequence $\{f_n\}$ is not uniformly convergent on $[0, 1]$. 4

- (b) Prove that the series $\sum \frac{x}{n+n^2x^2}$ is uniformly convergent for all real x . 4

9. (a) Show that the series $x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \dots$ is not uniformly convergent on $[0, 1]$. 4

- (b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be uniformly continuous on $\mathbb{R}$. For each $n \in \mathbb{N}$, let $f_n(x) = f(x + \frac{1}{n})$, $x \in \mathbb{R}$. Prove that the sequence $\{f_n\}$ is uniformly convergent on $\mathbb{R}$. 4

- 10.(a) If $\{u_n\}$ be a sequence of real numbers and $\sum u_n^2$ is convergent prove that $\sum \frac{u_n}{n}$ is absolutely convergent. 4

- (b) If $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences then prove that, 4

(i) $\{x_n + y_n\}$ is a Cauchy sequence

(ii) $\{x_n y_n\}$ is a Cauchy sequence.

N.B. : Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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