WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2021-22

MTMGDSE01T-MATHEMATICS (DSE1)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Express $v=(x,\,y)$ as a linear combination of $v_1=(1,1)$ and $v_2=(1,-1)$ in $\mathbb{R}^2$.

(b) What 2 by 2 matrices represent the transformations that

(i) rotate every point by an angle θ about the origin.

(ii) reflect every point about the *x*-axis.

(c) What is the geometric object corresponding to the smallest subspace V_0 containing a nonzero vector $v=(r,\,s,\,t)\in\mathbb{R}^3$? Answer with reason.

(d) Write the matrix equation for the system of equations:

$$x+y=3,\ -3y+4z=17,\ x-z=-8\,.$$

(e) Is there any straight line in the vector space R_2 which is a subspace of R_2?

(f) Find the inverse of the matrix $A=\begin{bmatrix}5 & 3\\ -2 & 2\end{bmatrix}$.

(g) For what values of *z* the three vectors (1, 1, 2), (*z*, 1, 1) and (1, 2, 1) are linearly independent?

(h) It is impossible for a system of linear equations to have exactly two solutions. Explain why.

(i) Prove that $\{(x,\,y,\,z)\in\mathbb{R}^3 : x^2+y^2=z^2\}$ is not a subspace of $\mathbb{R}^3$.

2. (a) Examine if the set *S* is a subspace of $\mathbf{R}_3$, $S=\{(x,\,y,\,z)\in\mathbf{R}_3 \mid x=0\ z=0\}$. 4

(b) If $\alpha=(1,\,2,\,0)$, $\beta=(3,-1,1)$, and $\gamma=(4,1,1)$, determine whether they are linearly dependent or not. 4

3. (a) If $A=\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ $B=\begin{bmatrix} p & q & r \\ s & t & u \end{bmatrix}$, $C=\begin{bmatrix} l & m \\ n & k \\ h & g \end{bmatrix}$, then establish that $(A+B)C=AC+BC$. 4

(b) If $P=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, and $Q=\begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix}$ then establish 4

(i) $(P+Q)^T=P^T+Q^T$ and (ii) $(P.Q)^T=Q^T.P^T$.

4. (a) Prove that two eigen vectors of a square matrix A over a field F corresponding to two distinct eigen values of A are linearly independent. 4

(b) Prove that the eigen values of a real symmetric matrix are all real. 4

5. (a) Prove that a matrix is non-singular if and only if it can be expressed as the product of a finite number of elementary matrices. 4

(b) Prove that if the rank of a real symmetric matrix be 1 then the diagonal elements of the matrix cannot be all zero. 4

6. (a) Diagonaliza the matrix $A=\begin{bmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{bmatrix}$. 5

(b) Define a basis of a vector space. Do the vectors (1, 1, 2), (3, 5, 2) and (1, 0, 0) form a basis of $\mathbb{R}^3$? Justify. 3

7. (a) Find the eigen vectors and eigenvalues of $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ 4

(b) If Q_θ represents the matrix for rotation (in x-y plane) through an angle θ about the origin, prove that $Q_\theta^2=Q_{2\theta}$ and $Q_\theta Q_{-\theta}=I_2$ 4

8. (a) State Cayley-Hamilton's Theorem and verify it for the matrix $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. 4

Hence find A^{-1}.

(b) What matrix has the effect of rotating every point through 90º and then projecting the result onto the x-axis? What matrix represents projection onto the x-axis followed by projection onto y-axis? 2+2

9. (a) Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{bmatrix}$. 4

(b) If $A=\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Correct or justify: 4

(i) $(A-B)(A+B)=A^2-B^2$

(ii) $(A-C)(A+C)=A^2-C^2$

10.(a) Express $A=\begin{bmatrix} 2 & 5 & -3 \\ 7 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ as a sum of a symmetric and skew symmetric matrix. 3

(b) If $A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $C=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ then verify that $AC=CA=6I_3$ and use this result to solve the system of equations 5

$x-y=3$, $2x+3y+4z=17$, $y+2z=7$.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2021-22

MTMGDSE02T-MATHEMATICS (DSE1)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

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Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Write down the conditions of equilibrium of a system of coplanar forces acting on a rigid body.

(b) Three forces *P*, *Q*, *R* act in the same sense along the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ of a triangle *ABC*. If their resultant passes through the in-centre then show that $P+Q+R=0$.

(c) Find the centre of gravity of the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

(d) A heavy body is in limiting equilibrium on a rough inclined plane under the action of gravity only, then what is the inclination of the plane with the horizontal?

(e) A particle moves along a straight line according to the law $s^2 = at^2 + bt + c$. Prove that its acceleration varies as $\frac{1}{s^3}$.

(f) At what height would the kinetic energy of a falling particle be equal to half of its potential energy?

(g) If a particle moves in a circle of radius *r* with uniform speed *v*, then find its angular velocity about the centre.

(h) A particle is projected under gravity at an angle α with the horizontal. Find the velocity of the particle at time *t*.

(i) A particle describes the curve $r = ae^{\theta}$ with constant angular velocity. Show that its radial acceleration is zero and the transverse acceleration varies as the distance from the pole.

2. Three forces *P*, *Q*, *R* act along the sides of the triangle formed by the lines $x+y=1$, $y-x=1$ and $y=2$. Find the equation of the line of action of their resultant. 8

3. Show that the least force which will move a weight W along a rough horizontal plane is $W\sin\phi$, where ϕ is the angle of friction. 8

4. A frustum of a cone is formed by cutting off the upper portion of a solid right circular cone by a plane parallel to the base. The radii of the parallel circular sections being R and r, and h the height of the frustum, show that the height of the centre of gravity of the frustum from the base is $\frac{h}{4}\cdot\frac{R^2+2Rr+3r^2}{R^2+Rr+r^2}$. 8

5. (a) A cycloid is placed with its axis vertical and vertex downwards. Show that a particle cannot rest at any point of the curve which is higher than $2a\sin^2\lambda$ above the lowest point, where λ is the angle of friction and a is the radius of the generating circle of the cycloid. 3

(b) Two equal uniform rods AB and AC, each of length $2b$ are freely jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them, then $b\sin^3\theta = a\cos\theta$. 5

6. (a) Deduce the expressions for tangential and normal components of the acceleration of a particle describing a plane curve. 5

(b) A particle describes a circle of radius a in such a way that its tangential acceleration is K times the normal acceleration, where K is a constant. If the speed of particle at any point be u, prove that it will return to the same point after a time 3

$$\frac{a}{Ku}(1-e^{-2\pi K})$$

7. Two particles are projected simultaneously from O in different directions with same speed u so as to pass through another point P. If α and β are the angles of projection prove that they pass through P at times separated by 8

$$\frac{2u}{g}\sin\frac{1}{2}(\alpha-\beta)\cdot\sec\frac{1}{2}(\alpha+\beta)$$

8. (a) A particle of mass m falls from rest at a distance a from the centre of force varying inversely as the square of the distance from the centre. Find the time it descends to the centre of force. 4

(b) A particle moving in a straight line starts from rest and the acceleration at any time t is $a-Kt^2$, where a and K are positive constant. Show that the maximum velocity attained by the particle is $\frac{2}{3}\sqrt{\frac{a^3}{K}}$. 4

9. (a) A particle rests in equilibrium under the attraction of two centre of force which attract directly as the distance, their attractions at unit distance being μ_1 and μ_2 respectively. The particle is slightly displaced towards one of the centres, show that the time of small oscillation is $\frac{2\pi}{\sqrt{\mu_1+\mu_2}}$. 4

(b) In a simple harmonic motion, if f be the acceleration and v be the velocity at any instant and T is periodic time, then show that $f^2T^4 + 4\pi^2v^2T^2 = 16\pi^4a^2$. 4

10.(a) A particle is projected vertically upwards with a velocity u in a medium whose resistance varies as the square of the velocity. Investigate the motion. 4

(b) If the radial and transverse velocities of a particle are $\mu\theta$ and λr respectively, show that the path of the particle can be represented by an equation of the form $r = A\theta^2 + B$. 4

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