WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2021

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours   Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Test whether the equation $xdx + ydy + \dfrac{xdy - ydx}{x^2 + y^2} = 0$ is exact or not.

(b) Find an integrating factor of the differential equation $(x \log x)\dfrac{dy}{dx} + y = 2\log x$.

(c) Find particular integral of the differential equation $2x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = \dfrac{1}{x}$.

(d) Find the transformation of the differential equation $x^2\dfrac{d^2y}{dx^2} - 5y = \log x$, using the substitution $x = e^z$.

(e) Find complementary function of the differential equation $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} = 3x$.

(f) Find the Wronskian of $y_1(x) = e^{-2x}$, $y_2(x) = xe^{-2x}$.

(g) Construct a PDE by eliminating a and b from $z = ae^{-b^2t}\cos bx$.

(h) Determine the order, degree and linearity of the following PDE:

$$\frac{\partial z}{\partial x} = \left(\frac{\partial^2 z}{\partial x^2}\right)^{5/2} + \left(\frac{\partial^2 z}{\partial y^2}\right)^{5/2}$$

(i) Classify the following PDE

$$(1+x^2)\, z_{xx} + (1+y^2)\, z_{yy} + xz_x + yz_y = 0$$

into elliptic, parabolic and hyperbolic for different values of x and y.

2. (a) Find an integrating factor of the differential equation 4

$$(2xy^4e^y + 2xy^3 + y)\, dx + (x^2y^4e^y - x^2y^2 - 3x)\, dy = 0$$

and hence solve it.

(b) Solve: $x\cos x\dfrac{dy}{dx}+y\,(x\sin x+\cos x)=1$ 4

3. (a) Find the curve for which the area of the triangle formed by x-axis, a tangent and the radius vector of the point of tangency is constant and equal to a^2. 4

(b) Using the substitution $u=\dfrac{1}{x}$ and $v=\dfrac{1}{y}$, reduce the equation $y^2(y-px)=x^4p^2$ to Clairaut's form and hence solve it. Here $p\equiv\dfrac{dy}{dx}$. 4

4. (a) Show that each of the functions e^x, e^{4x} and $2e^x-3e^{4x}$ is solution of the differential equation $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+4y=0,\ -\infty<x<\infty$. 2+1+1+1

Are the three independent? If not, find which two of these are independent. Write down a general solution of the equation.

(b) Find the value of h so that the equation $(ax+hy+g)\,dx+(3x+by+f)\,dy=0$ becomes an exact differential equation. 3

5. (a) Solve by the method of variation of parameters: 5

$$(D^2-3D+2)y=e^x(1+e^x)^{-1},\text{ where } D\equiv\frac{d}{dx}$$

(b) Find particular integral of the differential equation 3

$$(D^2+5D+6)y=e^{-2x}\sin 2x,\quad\text{where } D\equiv\frac{d}{dx}$$

6. (a) Solve in the particular cases: 5

$$\frac{d^2x}{dt^2}-4\frac{dx}{dt}+5x=0 \text{ giving that } x=1 \text{ and } \frac{dx}{dt}=2 \text{ when } x=0$$

(b) Solve: $\dfrac{d^2y}{dx^2}=x^2\sin x$ 3

7. (a) Solve the following total differential equation: 4

$$yz\,dx+2zx\,dy-3xy\,dz=0$$

(b) Solve: $x^2\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}+y=x\log x$ 4

8. (a) Form a PDE by eliminating the arbitrary function ϕ from 4

$$lx+my+nz=\phi(x^2+y^2+z^2)$$

(b) Solve the partial differential equation by Lagrange's method $x^2p+y^2q=(x+y)z$. 4

9. (a) Find the partial differential equation of planes having equal intercepts along x axis and y axis. 4

(b) Find $f(y)$ such that the total differential equation $\left(\frac{yz+z}{x}\right)dx - zdy + f(y)\,dz = 0$ is integrable. 4

10.(a) Formulate a PDE from the relation $f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0$. 3

(b) Find the Wronskian of x and $|x|$ in $[-1, 1]$. 2

(c) Solve $x^2 \frac{d^2y}{dx^2} - 6y = 0$. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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